1D nonlinear Fokker-Planck equations for fermions and bosons

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We analyze the large-time behavior of solutions of the Cauchy problem:

\[
\begin{aligned}
\frac{\partial f}{\partial t} &= \frac{\partial^2}{\partial v^2} f + \frac{\partial}{\partial v} [v f(1 + k f)], \\
 f(v, 0) &= f_0(v)
\end{aligned}
\]  

(1)

These nonlinear Fokker-Planck equations have been proposed in [?, ?] and the references therein, as kinetic models for the relaxation to equilibrium for bosons \((k = 1)\) and fermions \((k = -1)\). These models have been introduced as a simplification with respect to Boltzmann-based models as in [?]. Here, we will show that entropy methods apply in a direct way to analyze the equilibration rate for the one-dimensional case. The entropy functional we use coincides with the one introduced in [?] for the nonlinear diffusion equation

\[
\frac{\partial g}{\partial t} = \frac{\partial}{\partial x} \left\{ g \frac{\partial}{\partial x} \left[ x + \log \left( \frac{g}{1 + k g} \right) \right] \right\}
\]  

(2)

for a function \(g(x, t), x \in \mathbb{R}, t > 0\). The relation between the entropy dissipations for the solutions of the nonlinear diffusion equation (2) and (1) will be the basis of our proof of an exponential decay rate.


